# CHAPTER 5
# Stoichiometry and the Mole

## Opening Essay

At Contrived State University in Anytown, Ohio, a new building was dedicated in March 2010 to house the College of Education. The 100,000-square-foot building has enough office space to accommodate 86 full-time faculty members and 167 full-time staff.

In a fit of monetary excess, the university administration offered to buy new furniture (desks and chairs) and computer workstations for all faculty and staff members moving into the new building. However, to save on long-term energy and materials costs, the university offered to buy only 1 laser printer per 10 employees, with the plan to network the printers together.

How many laser printers did the administration have to buy? It is rather simple to show that 26 laser printers are needed for all the employees. However, what if a chemist was calculating quantities for a chemical reaction? Interestingly enough, similar calculations can be performed for chemicals as well as laser printers.

**Outfitting a New Building**
In filling a new office building with furniture and equipment, managers do calculations similar to those performed by scientists doing chemical reactions.

*Source: Photo courtesy of Benjamin Benschneider, Cleveland State University.*

We have already established that quantities are important in science, especially in chemistry. It is important to make accurate measurements of a variety of quantities when performing experiments. However, it is also important to be able to relate one measured quantity to another, unmeasured quantity. In this chapter, we will consider how we manipulate quantities to relate them to each other.

# 1. STOICHIOMETRY

### LEARNING OBJECTIVES

1. Define *stoichiometry*.
2. Relate quantities in a balanced chemical reaction on a molecular basis.

Consider a classic recipe for pound cake: 1 pound of eggs, 1 pound of butter, 1 pound of flour, and 1 pound of sugar. (That's why it's called "pound cake.") If you have 4 pounds of butter, how many pounds of sugar, flour, and eggs do you need? You would need 4 pounds each of sugar, flour, and eggs.

Now suppose you have 1.00 g $H_2$. If the chemical reaction follows the balanced chemical equation

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$$

then what mass of oxygen do you need to make water?

**stoichiometry**

The relating of one chemical substance to another using a balanced chemical reaction.

Curiously, this chemical reaction question is very similar to the pound cake question. Both of them involve relating a quantity of one substance to a quantity of another substance or substances. The relating of one chemical substance to another using a balanced chemical reaction is called **stoichiometry**. Using stoichiometry is a fundamental skill in chemistry; it greatly broadens your ability to predict what will occur and, more importantly, how much is produced.

Let us consider a more complicated example. A recipe for pancakes calls for 2 cups (c) of pancake mix, 1 egg, and 1/2 c of milk. We can write this in the form of a chemical equation:

$$2 \text{ c mix} + 1 \text{ egg} + 1/2 \text{ c milk} \rightarrow 1 \text{ batch of pancakes}$$

If you have 9 c of pancake mix, how many eggs and how much milk do you need? It might take a little bit of work, but eventually you will find you need 4½ eggs and 2¼ c milk.

How can we formalize this? We can make a conversion factor using our original recipe and use that conversion factor to convert from a quantity of one substance to a quantity of another substance, similar to the way we constructed a conversion factor between feet and yards in Chapter 2. Because one recipe's worth of pancakes requires 2 c of pancake mix, 1 egg, and 1/2 c of milk, we actually have the following mathematical relationships that relate these quantities:

$$2 \text{ c pancake mix} \Leftrightarrow 1 \text{ egg} \Leftrightarrow 1/2 \text{ c milk}$$

where ⇔ is the mathematical symbol for "is equivalent to." This does not mean that 2 c of pancake mix equal 1 egg. However, *as far as this recipe is concerned*, these are the equivalent quantities needed for a single recipe of pancakes. So, any possible quantities of two or more ingredients must have the same numerical ratio as the ratios in the equivalence.

We can deal with these equivalences in the same way we deal with equalities in unit conversions: we can make conversion factors that essentially equal 1. For example, to determine how many eggs we need for 9 c of pancake mix, we construct the conversion factor

$$\frac{1 \text{ egg}}{2 \text{ c pancake mix}}$$

This conversion factor is, in a strange way, equivalent to 1 because the recipe relates the two quantities. Starting with our initial quantity and multiplying by our conversion factor,

$$9 \cancel{\text{ c pancake mix}} \times \frac{1 \text{ egg}}{2 \cancel{\text{ c pancake mix}}} = 4.5 \text{ eggs}$$

Note how the units *cups pancake mix* canceled, leaving us with units of *eggs*. This is the formal, mathematical way of getting our amounts to mix with 9 c of pancake mix. We can use a similar conversion factor for the amount of milk:

$$9 \cancel{\text{ c pancake mix}} \times \frac{1/2 \text{ c milk}}{2 \cancel{\text{ c pancake mix}}} = 2.25 \text{ c milk}$$

Again, units cancel, and new units are introduced.

A balanced chemical equation is nothing more than *a recipe for a chemical reaction*. The difference is that a balanced chemical equation is written in terms of atoms and molecules, not cups, pounds, and eggs.

For example, consider the following chemical equation:

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$$

We can interpret this as, literally, "two hydrogen molecules react with one oxygen molecule to make two water molecules." That interpretation leads us directly to some equivalences, just as our pancake recipe did:

$$2H_2 \text{ molecules} \Leftrightarrow 1O_2 \text{ molecule} \Leftrightarrow 2H_2O \text{ molecules}$$

These equivalences allow us to construct conversion factors:

$$\frac{2 \text{ molecules } H_2}{1 \text{ molecule } O_2} \quad \frac{2 \text{ molecules } H_2}{2 \text{ molecules } H_2O} \quad \frac{1 \text{ molecule } O_2}{2 \text{ molecules } H_2O}$$

and so forth. These conversions can be used to relate quantities of one substance to quantities of another. For example, suppose we need to know how many molecules of oxygen are needed to react with 16 molecules of $H_2$. As we did with converting units, we start with our given quantity and use the appropriate conversion factor:

$$16 \cancel{\text{ molecules } H_2} \times \frac{1 \text{ molecule } O_2}{2 \cancel{\text{ molecules } H_2}} = 8 \text{ molecules } O_2$$

Note how the unit *molecules* $H_2$ cancels algebraically, just as any unit does in a conversion like this. The conversion factor came directly from the coefficients in the balanced chemical equation. This is another reason why a properly balanced chemical equation is important.

## EXAMPLE 1

How many molecules of $SO_3$ are needed to react with 144 molecules of $Fe_2O_3$ given this balanced chemical equation?

$$Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$$

**Solution**

We use the balanced chemical equation to construct a conversion factor between $Fe_2O_3$ and $SO_3$. The number of molecules of $Fe_2O_3$ goes on the bottom of our conversion factor so it cancels with our given amount, and the molecules of $SO_3$ go on the top. Thus, the appropriate conversion factor is

$$\frac{3 \text{ molecules } SO_3}{1 \text{ molecule } Fe_2O_3}$$

Starting with our given amount and applying the conversion factor, the result is

$$144 \cancel{\text{ molecules } Fe_2O_3} \times \frac{3 \text{ molecules } SO_3}{1 \cancel{\text{ molecule } Fe_2O_3}} = 432 \text{ molecules } SO_3$$

We need 432 molecules of $SO_3$ to react with 144 molecules of $Fe_2O_3$.

***Test Yourself***

How many molecules of $H_2$ are needed to react with 29 molecules of $N_2$ to make ammonia if the balanced chemical equation is $N_2 + 3H_2 \rightarrow 2NH_3$?

*Answer*

87 molecules

Chemical equations also allow us to make conversions regarding the number of atoms in a chemical reaction because a chemical formula lists the number of atoms of each element in a compound. The formula $H_2O$ indicates that there are two hydrogen atoms and one oxygen atom in each molecule, and these relationships can be used to make conversion factors:

$$\frac{2 \text{ atoms H}}{1 \text{ molecule } H_2O} \quad \frac{1 \text{ molecule } H_2O}{1 \text{ atom O}}$$

Conversion factors like this can also be used in stoichiometry calculations.

## EXAMPLE 2

How many molecules of $NH_3$ can you make if you have 228 atoms of $H_2$?

**Solution**

From the formula, we know that one molecule of $NH_3$ has three H atoms. Use that fact as a conversion factor:

$$228\ \cancel{\text{atoms H}} \times \frac{1\ \text{molecule NH}_3}{3\ \cancel{\text{atoms H}}} = 76\ \text{molecules NH}_3$$

***Test Yourself***

How many molecules of $Fe_2(SO_4)_3$ can you make from 777 atoms of S?

*Answer*

259 molecules

## KEY TAKEAWAY

- Quantities of substances can be related to each other using balanced chemical equations.

## EXERCISES

1. Think back to the pound cake recipe. What possible conversion factors can you construct relating the components of the recipe?
2. Think back to the pancake recipe. What possible conversion factors can you construct relating the components of the recipe?
3. What are all the conversion factors that can be constructed from the balanced chemical reaction $2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$?
4. What are all the conversion factors that can be constructed from the balanced chemical reaction $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$?
5. Given the chemical equation

   $$Na(s) + H_2O(\ell) \rightarrow NaOH(aq) + H_2(g)$$

   a. Balance the equation.
   b. How many molecules of $H_2$ are produced when 332 atoms of Na react?
6. Given the chemical equation

   $$S(s) + O_2(g) \rightarrow SO_3(g)$$

   a. Balance the equation.
   b. How many molecules of $O_2$ are needed when 38 atoms of S react?
7. For the balanced chemical equation

   $$6H^+(aq) + 2MnO_4^-(aq) + 5H_2O_2(\ell) \rightarrow 2Mn^{2+}(aq) + 5O_2(g) + 8H_2O(\ell)$$

   how many molecules of $H_2O$ are produced when 75 molecules of $H_2O_2$ react?
8. For the balanced chemical reaction

   $$2C_6H_6(\ell) + 15O_2(g) \rightarrow 12CO_2(g) + 6H_2O(\ell)$$

   how many molecules of $CO_2$ are produced when 56 molecules of $C_6H_6$ react?
9. Given the balanced chemical equation

   $$Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$$

   how many molecules of $Fe_2(SO_4)_3$ are produced if 321 atoms of S are reacted?
10. For the balanced chemical equation

    $$CuO(s) + H_2S(g) \rightarrow CuS + H_2O(\ell)$$

    how many molecules of CuS are formed if 9,044 atoms of H react?
11. For the balanced chemical equation

    $$Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$$

    suppose we need to make 145,000 molecules of $Fe_2(SO_4)_3$. How many molecules of $SO_3$ do we need?
12. One way to make sulfur hexafluoride is to react thioformaldehyde, $CH_2S$, with elemental fluorine:

    $$CH_2S + 6F_2 \rightarrow CF_4 + 2HF + SF_6$$

    If 45,750 molecules of $SF_6$ are needed, how many molecules of $F_2$ are required?

13. Construct the three independent conversion factors possible for these two reactions:
    a. $2H_2 + O_2 \rightarrow 2H_2O$
    b. $H_2 + O_2 \rightarrow H_2O_2$

    Why are the ratios between $H_2$ and $O_2$ different?

    The conversion factors are different because the stoichiometries of the balanced chemical reactions are different.

14. Construct the three independent conversion factors possible for these two reactions:
    a. $2Na + Cl_2 \rightarrow 2NaCl$
    b. $4Na + 2Cl_2 \rightarrow 4NaCl$

    What similarities, if any, exist in the conversion factors from these two reactions?

**ANSWERS**

1. $\frac{\text{1 pound butter}}{\text{1 pound flour}}$ or $\frac{\text{1 pound sugar}}{\text{1 pound eggs}}$ are two conversion factors that can be constructed from the pound cake recipe. Other conversion factors are also possible.

3. $\frac{\text{2 molecules } H_2}{\text{1 molecule } O_2}$, $\frac{\text{1 molecule } O_2}{\text{2 molecules } H_2O}$, $\frac{\text{2 molecules } H_2}{\text{2 molecules } H_2O}$, and their reciprocals are the conversion factors that can be constructed.

5. a. $2Na(s) + 2H_2O(\ell) \rightarrow 2NaOH(aq) + H_2(g)$
   b. 166 molecules

7. 120 molecules

9. 107 molecules

11. 435,000 molecules

13. a. $\frac{\text{2 molecules } H_2}{\text{1 molecule } O_2}$, $\frac{\text{1 molecule } O_2}{\text{2 molecules } H_2O}$, and $\frac{\text{2 molecules } H_2}{\text{2 molecules } H_2O}$
    b. $\frac{\text{1 molecule } H_2}{\text{1 molecule } O_2}$, $\frac{\text{1 molecule } O_2}{\text{1 molecule } H_2O_2}$, and $\frac{\text{1 molecule } H_2}{\text{1 molecule } H_2O_2}$

## 2. THE MOLE

**LEARNING OBJECTIVES**

1. **Describe the unit *mole*.**
2. **Relate the mole quantity of substance to its mass.**

So far, we have been talking about chemical substances in terms of individual atoms and molecules. Yet we don't typically deal with substances an atom or a molecule at a time; we work with millions, billions, and trillions of atoms and molecules at a time. What we need is a way to deal with macroscopic, rather than microscopic, amounts of matter. We need a unit of amount that relates quantities of substances on a scale that we can interact with.

Chemistry uses a unit called mole. A **mole** (mol) is a number of things equal to the number of atoms in exactly 12 g of carbon-12. Experimental measurements have determined that this number is very large:

$$1 \text{ mol} = 6.02214179 \times 10^{23} \text{ things}$$

**mole**

The number of things equal to the number of atoms in exactly 12 g of carbon-12; equals $6.022 \times 10^{23}$ things.

Understand that a mole means a number of things, just like a dozen means a certain number of things—twelve, in the case of a dozen. But a mole is a much larger number of things. These things can be atoms, or molecules, or eggs; however, in chemistry, we usually use the mole to refer to the amounts of atoms or molecules. Although the number of things in a mole is known to eight decimal places, it is usually fine to use only two or three decimal places in calculations. The numerical value of things in a mole is often called *Avogadro's number* ($N_A$), which is also known as the *Avogadro constant*, after Amadeo Avogadro, an Italian chemist who first proposed its importance.

## EXAMPLE 3

How many molecules are present in 2.76 mol of $H_2O$? How many atoms is this?

**Solution**

The definition of a mole is an equality that can be used to construct a conversion factor. Also, because we know that there are three atoms in each molecule of $H_2O$, we can also determine the number of atoms in the sample.

$$2.76\ \cancel{\text{mol } H_2O} \times \frac{6.022 \times 10^{23} \text{molecules } H_2O}{\cancel{\text{mol } H_2O}} = 1.66 \times 10^{24} \text{ molecules } H_2O$$

To determine the total number of atoms, we have

$$1.66 \times 10^{24}\ \cancel{\text{molecules } H_2O} \times \frac{3 \text{ atoms}}{1\ \cancel{\text{molecule}}} = 4.99 \times 10^{24} \text{ atoms}$$

***Test Yourself***

How many molecules are present in $4.61 \times 10^{-2}$ mol of $O_2$?

*Answer*

$2.78 \times 10^{22}$ molecules

How big is a mole? It is very large. Suppose you had a mole of dollar bills that need to be counted. If everyone on earth (about 6 billion people) counted one bill per second, it would take about 3.2 million years to count all the bills. A mole of sand would fill a cube about 32 km on a side. A mole of pennies stacked on top of each other would have about the same diameter as our galaxy, the Milky Way. A mole is a lot of things—but atoms and molecules are very tiny. One mole of carbon atoms would make a cube that is 1.74 cm on a side, small enough to carry in your pocket.

Why is the mole unit so important? It represents the link between the microscopic and the macroscopic, especially in terms of mass. *A mole of a substance has the same mass in grams as one unit (atom or molecules) has in atomic mass units.* The mole unit allows us to express amounts of atoms and molecules in visible amounts that we can understand.

For example, we already know that, by definition, a mole of carbon has a mass of exactly 12 g. This means that exactly 12 g of C has $6.022 \times 10^{23}$ atoms:

$$12 \text{ g C} = 6.022 \times 10^{23} \text{ atoms C}$$

We can use this equality as a conversion factor between the number of atoms of carbon and the number of grams of carbon. How many grams are there, say, in $1.50 \times 10^{25}$ atoms of carbon? This is a one-step conversion:

$$1.50 \times 10^{25}\ \cancel{\text{atoms C}} \times \frac{12.0000 \text{ g C}}{6.022 \times 10^{23}\ \cancel{\text{atoms C}}} = 299 \text{ g C}$$

**molar mass**

The mass of 1 mol of a substance in grams.

But it also goes beyond carbon. Previously we defined atomic and molecular masses as the number of atomic mass units per atom or molecule. Now we can do so in terms of grams. The atomic mass of an element is the number of grams in 1 mol of atoms of that element, while the molecular mass of a compound is the number of grams in 1 mol of molecules of that compound. Sometimes these masses are called **molar masses** to emphasize the fact that they are the mass for 1 mol of things. (The term *molar* is the adjective form of mole and has nothing to do with teeth.)

Here are some examples. The mass of a hydrogen atom is 1.0079 u; the mass of 1 mol of hydrogen atoms is 1.0079 g. Elemental hydrogen exists as a diatomic molecule, $H_2$. One molecule has a mass of 1.0079 + 1.0079 = 2.0158 u, while 1 mol $H_2$ has a mass of 2.0158 g. A molecule of $H_2O$ has a mass of about 18.01 u; 1 mol $H_2O$ has a mass of 18.01 g. A single unit of NaCl has a mass of 58.45 u; NaCl has a molar mass of 58.45 g. In each of these moles of substances, there are $6.022 \times 10^{23}$ units: $6.022 \times 10^{23}$ atoms of H, $6.022 \times 10^{23}$ molecules of $H_2$ and $H_2O$, $6.022 \times 10^{23}$ units of NaCl ions. These relationships give us plenty of opportunities to construct conversion factors for simple calculations.

## EXAMPLE 4

What is the molar mass of $C_6H_{12}O_6$?

**Solution**

To determine the molar mass, we simply add the atomic masses of the atoms in the molecular formula but express the total in grams per mole, not atomic mass units. The masses of the atoms can be taken from the periodic table or the list of elements in Chapter 17:

| | |
|---|---|
| 6 C = 6 × 12.011 | = 72.066 |
| 12 H = 12 × 1.0079 | = 12.0948 |
| 6 O = 6 × 15.999 | = 95.994 |
| TOTAL | = 180.155 g/mol |

Per convention, the unit *grams per mole* is written as a fraction.

***Test Yourself***

What is the molar mass of $AgNO_3$?

*Answer*

169.87 g/mol

Knowing the molar mass of a substance, we can calculate the number of moles in a certain mass of a substance and vice versa, as these examples illustrate. The molar mass is used as the conversion factor.

## EXAMPLE 5

What is the mass of 3.56 mol of $HgCl_2$? The molar mass of $HgCl_2$ is 271.49 g/mol.

**Solution**

Use the molar mass as a conversion factor between moles and grams. Because we want to cancel the mole unit and introduce the gram unit, we can use the molar mass as given:

$$3.56\ \cancel{\text{mol HgCl}_2} \times \frac{271.49\ \text{g HgCl}_2}{\cancel{\text{mol HgCl}_2}} = 967\ \text{g HgCl}_2$$

***Test Yourself***

What is the mass of 33.7 mol of $H_2O$?

*Answer*

607 g

## EXAMPLE 6

How many moles of $H_2O$ are present in 240.0 g of water (about the mass of a cup of water)?

**Solution**

Use the molar mass of $H_2O$ as a conversion factor from mass to moles. The molar mass of water is (1.0079 + 1.0079 + 15.999) = 18.015 g/mol. However, because we want to cancel the gram unit and introduce moles, we need to take the reciprocal of this quantity, or 1 mol/18.015 g:

$$240.0\ \cancel{\text{g H}_2\text{O}} \times \frac{1\ \text{mol H}_2\text{O}}{18.015\ \cancel{\text{g H}_2\text{O}}} = 13.32\ \text{mol H}_2\text{O}$$

***Test Yourself***

How many moles are present in 35.6 g of $H_2SO_4$ (molar mass = 98.08 g/mol)?

*Answer*

0.363 mol

Other conversion factors can be combined with the definition of mole—density, for example.

### EXAMPLE 7

The density of ethanol is 0.789 g/mL. How many moles are in 100.0 mL of ethanol? The molar mass of ethanol is 46.08 g/mol.

**Solution**

Here, we use density to convert from volume to mass and then use the molar mass to determine the number of moles.

$$100.0\ \cancel{\text{mL}}\ \text{ethanol} \times \frac{0.789\ \cancel{\text{g}}}{\cancel{\text{mL}}} \times \frac{1\ \text{mol}}{46.08\ \cancel{\text{g}}} = 1.71\ \text{mol ethanol}$$

***Test Yourself***

If the density of benzene, $C_6H_6$, is 0.879 g/mL, how many moles are present in 17.9 mL of benzene?

*Answer*

0.201 mol

### KEY TAKEAWAYS

- The mole is a key unit in chemistry.
- The molar mass of a substance, in grams, is numerically equal to one atom's or molecule's mass in atomic mass units.

### EXERCISES

1. How many atoms are present in 4.55 mol of Fe?
2. How many atoms are present in 0.0665 mol of K?
3. How many molecules are present in 2.509 mol of $H_2S$?
4. How many molecules are present in 0.336 mol of acetylene ($C_2H_2$)?
5. How many moles are present in $3.55 \times 10^{24}$ Pb atoms?
6. How many moles are present in $2.09 \times 10^{22}$ Ti atoms?
7. How many moles are present in $1.00 \times 10^{23}$ $PF_3$ molecules?
8. How many moles are present in $5.52 \times 10^{25}$ penicillin molecules?
9. Determine the molar mass of each substance.
   a. Si
   b. $SiH_4$
   c. $K_2O$
10. Determine the molar mass of each substance.
    a. $Cl_2$
    b. $SeCl_2$
    c. $Ca(C_2H_3O_2)_2$
11. Determine the molar mass of each substance.
    a. Al
    b. $Al_2O_3$
    c. $CoCl_3$
12. Determine the molar mass of each substance.
    a. $O_3$
    b. NaI
    c. $C_{12}H_{22}O_{11}$
13. What is the mass of 4.44 mol of Rb?
14. What is the mass of 0.311 mol of Xe?
15. What is the mass of 12.34 mol of $Al_2(SO_4)_3$?
16. What is the mass of 0.0656 mol of $PbCl_2$?
17. How many moles are present in 45.6 g of CO?
18. How many moles are present in 0.00339 g of LiF?
19. How many moles are present in 1.223 g of $SF_6$?
20. How many moles are present in 48.8 g of $BaCO_3$?
21. How many moles are present in 54.8 mL of mercury if the density of mercury is 13.6 g/mL?
22. How many moles are present in 56.83 mL of $O_2$ if the density of $O_2$ is 0.00133 g/mL?

**ANSWERS**

1. $2.74 \times 10^{24}$ atoms
3. $1.511 \times 10^{24}$ molecules
5. 5.90 mol
7. 0.166 mol
9. a. 28.086 g
   b. 32.118 g
   c. 94.195 g
11. a. 26.981 g
    b. 101.959 g
    c. 165.292 g
13. 379 g
15. 4,222 g
17. 1.63 mol
19. 0.008374 mol
21. 3.72 mol

# 3. THE MOLE IN CHEMICAL REACTIONS

**LEARNING OBJECTIVES**

1. **Balance a chemical equation in terms of moles.**
2. **Use the balanced equation to construct conversion factors in terms of moles.**
3. **Calculate moles of one substance from moles of another substance using a balanced chemical equation.**

Consider this balanced chemical equation:

$$2H_2 + O_2 \rightarrow 2H_2O$$

We interpret this as "two molecules of hydrogen react with one molecule of oxygen to make two molecules of water." The chemical equation is balanced as long as the coefficients are in the ratio 2:1:2. For instance, this chemical equation is also balanced:

$$100H_2 + 50O_2 \rightarrow 100H_2O$$

This equation is not conventional—because convention says that we use the lowest ratio of coefficients—but it is balanced. So is this chemical equation:

$$5{,}000H_2 + 2{,}500O_2 \rightarrow 5{,}000H_2O$$

Again, this is not conventional, but it is still balanced. Suppose we use a much larger number:

$$12.044 \times 10^{23}\ H_2 + 6.022 \times 10^{23}\ O_2 \rightarrow 12.044 \times 10^{23}\ H_2O$$

These coefficients are also in the ratio of 2:1:2. But these numbers are related to the number of things in a mole: the first and last numbers are two times Avogadro's number, while the second number is Avogadro's number. That means that the first and last numbers represent 2 mol, while the middle number is just 1 mol. Well, why not just use the number of moles in balancing the chemical equation?

$$2H_2 + O_2 \rightarrow 2H_2O$$

is the same balanced chemical equation we started with! What this means is that chemical equations are not just balanced in terms of molecules; *they are also balanced in terms of moles*. We can just as easily read this chemical equation as "two moles of hydrogen react with one mole of oxygen to make two moles of water." All balanced chemical reactions are balanced in terms of moles.

### EXAMPLE 8

Interpret this balanced chemical equation in terms of moles.

$$P_4 + 5O_2 \rightarrow P_4O_{10}$$

**Solution**

The coefficients represent the number of moles that react, not just molecules. We would speak of this equation as "one mole of molecular phosphorus reacts with five moles of elemental oxygen to make one mole of tetraphosphorus decoxide."

***Test Yourself***

Interpret this balanced chemical equation in terms of moles.

$$N_2 + 3H_2 \rightarrow 2NH_3$$

*Answer*

One mole of elemental nitrogen reacts with three moles of elemental hydrogen to produce two moles of ammonia.

In Chapter 4, Section 1, we stated that a chemical equation is simply a recipe for a chemical reaction. As such, chemical equations also give us equivalences—equivalences between the reactants and the products. However, now we understand that *these equivalences are expressed in terms of moles*. Consider the chemical equation

$$2H_2 + O_2 \rightarrow 2H_2O$$

This chemical reaction gives us the following equivalences:

$$2 \text{ mol } H_2 \Leftrightarrow 1 \text{ mol } O_2 \Leftrightarrow 2 \text{ mol } H_2O$$

Any two of these quantities can be used to construct a conversion factor that lets us relate the number of moles of one substance to an equivalent number of moles of another substance. If, for example, we want to know how many moles of oxygen will react with 17.6 mol of hydrogen, we construct a conversion factor between 2 mol of $H_2$ and 1 mol of $O_2$ and use it to convert from moles of one substance to moles of another:

$$17.6 \cancel{\text{mol } H_2} \times \frac{1 \text{ mol } O_2}{2 \cancel{\text{mol } H_2}} = 8.80 \text{ mol } O_2$$

Note how the mol $H_2$ unit cancels, and mol $O_2$ is the new unit introduced. This is an example of a **mole-mole calculation**, when you start with moles of one substance and convert to moles of another substance by using the balanced chemical equation. The example may seem simple because the numbers are small, but numbers won't always be so simple!

**mole-mole calculation**

A stoichiometry calculation when one starts with moles of one substance and convert to moles of another substance using the balanced chemical equation.

## EXAMPLE 9

For the balanced chemical equation

$$2C_4H_{10}(g) + 13O_2 \rightarrow 8CO_2(g) + 10H_2O(\ell)$$

if 154 mol of $O_2$ are reacted, how many moles of $CO_2$ are produced?

**Solution**

We are relating an amount of oxygen to an amount of carbon dioxide, so we need the equivalence between these two substances. According to the balanced chemical equation, the equivalence is

$$13 \text{ mol } O_2 \Leftrightarrow 8 \text{ mol } CO_2$$

We can use this equivalence to construct the proper conversion factor. We start with what we are given and apply the conversion factor:

$$154 \cancel{\text{mol } O_2} \times \frac{8 \text{ mol } CO_2}{13 \cancel{\text{mol } O_2}} = 94.8 \text{ mol } CO_2$$

The mol $O_2$ unit is in the denominator of the conversion factor so it cancels. Both the 8 and the 13 are exact numbers, so they don't contribute to the number of significant figures in the final answer.

***Test Yourself***

Using the above equation, how many moles of $H_2O$ are produced when 154 mol of $O_2$ react?

*Answer*

118 mol

It is important to reiterate that balanced chemical equations are balanced in terms of *moles*. Not grams, kilograms, or liters—but moles. Any stoichiometry problem will likely need to work through the mole unit at some point, especially if you are working with a balanced chemical reaction.

## KEY TAKEAWAYS

- Balanced chemical reactions are balanced in terms of moles.
- A balanced chemical reaction gives equivalences in moles that allow stoichiometry calculations to be performed.

## EXERCISES

1. Express in mole terms what this chemical equation means.

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

2. Express in mole terms what this chemical equation means.

$$Na_2CO_3 + 2HCl \rightarrow 2NaCl + H_2O + CO_2$$

3. How many molecules of each substance are involved in the equation in Exercise 1 if it is interpreted in terms of moles?
4. How many molecules of each substance are involved in the equation in Exercise 2 if it is interpreted in terms of moles?
5. For the chemical equation

$$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$$

what equivalences can you write in terms of moles? Use the ⇔ sign.

6. For the chemical equation

$$2Al + 3Cl_2 \rightarrow 2AlCl_3$$

what equivalences can you write in terms of moles? Use the ⇔ sign.

7. Write the balanced chemical reaction for the combustion of $C_5H_{12}$ (the products are $CO_2$ and $H_2O$) and determine how many moles of $H_2O$ are formed when 5.8 mol of $O_2$ are reacted.

8. Write the balanced chemical reaction for the formation of $Fe_2(SO_4)_3$ from $Fe_2O_3$ and $SO_3$ and determine how many moles of $Fe_2(SO_4)_3$ are formed when 12.7 mol of $SO_3$ are reacted.
9. For the balanced chemical equation

$$3Cu(s) + 2NO_3^{-}(aq) + 8H^{+}(aq) \rightarrow 3Cu^{2+}(aq) + 4H_2O(\ell) + 2NO(g)$$

how many moles of $Cu^{2+}$ are formed when 55.7 mol of $H^+$ are reacted?
10. For the balanced chemical equation

$$Al(s) + 3Ag^{+}(aq) \rightarrow Al^{3+}(aq) + 3Ag(s)$$

how many moles of Ag are produced when 0.661 mol of Al are reacted?
11. For the balanced chemical reaction

$$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(\ell)$$

how many moles of $H_2O$ are produced when 0.669 mol of $NH_3$ react?
12. For the balanced chemical reaction

$$4NaOH(aq) + 2S(s) + 3O_2(g) \rightarrow 2Na_2SO_4(aq) + 2H_2O(\ell)$$

how many moles of $Na_2SO_4$ are formed when 1.22 mol of $O_2$ react?
13. For the balanced chemical reaction

$$4KO_2(s) + 2CO_2(g) \rightarrow 2K_2CO_3(s) + 3O_2(g)$$

determine the number of moles of both products formed when 6.88 mol of $KO_2$ react.
14. For the balanced chemical reaction

$$2AlCl_3 + 3H_2O(\ell) \rightarrow Al_2O_3 + 6HCl(g)$$

determine the number of moles of both products formed when 0.0552 mol of $AlCl_3$ react.

**ANSWERS**

1. One mole of $CH_4$ reacts with 2 mol of $O_2$ to make 1 mol of $CO_2$ and 2 mol of $H_2O$.
3. $6.022 \times 10^{23}$ molecules of $CH_4$, $1.2044 \times 10^{24}$ molecules of $O_2$, $6.022 \times 10^{23}$ molecules of $CO_2$, and $1.2044 \times 10^{24}$ molecules of $H_2O$
5. 2 mol of $C_2H_6$ ⇔ 7 mol of $O_2$ ⇔ 4 mol of $CO_2$ ⇔ 6 mol of $H_2O$
7. $C_5H_{12} + 8O_2 \rightarrow 5CO_2 + 6H_2O$; 4.4 mol
9. 20.9 mol
11. 1.00 mol
13. 3.44 mol of $K_2CO_3$; 5.16 mol of $O_2$

# 4. MOLE-MASS AND MASS-MASS CALCULATIONS

**LEARNING OBJECTIVES**

1. **From a given number of moles of a substance, calculate the mass of another substance involved using the balanced chemical equation.**
2. **From a given mass of a substance, calculate the moles of another substance involved using the balanced chemical equation.**
3. **From a given mass of a substance, calculate the mass of another substance involved using the balanced chemical equation.**

**mole-mass calculation**

A calculation in which you start with a given number of moles of a substance and calculate the mass of another substance involved in the chemical equation, or vice versa.

Mole-mole calculations are not the only type of calculations that can be performed using balanced chemical equations. Recall that the molar mass can be determined from a chemical formula and used as a conversion factor. We can add that conversion factor as another step in a calculation to make a **mole-mass calculation**, where we start with a given number of moles of a substance and calculate the mass of another substance involved in the chemical equation, or vice versa.

For example, suppose we have the balanced chemical equation

$$2Al + 3Cl_2 \rightarrow 2AlCl_3$$

Suppose we know we have 123.2 g of $Cl_2$. How can we determine how many moles of $AlCl_3$ we will get when the reaction is complete? First and foremost, *chemical equations are not balanced in terms of grams; they are balanced in terms of moles*. So to use the balanced chemical equation to relate an amount of $Cl_2$ to an amount of $AlCl_3$, we need to convert the given amount of $Cl_2$ into moles. We know how to do this by simply using the molar mass of $Cl_2$ as a conversion factor. The molar mass of $Cl_2$ (which we get from the atomic mass of Cl from the periodic table) is 70.90 g/mol. We must invert this fraction so that the units cancel properly:

$$123.2\ \cancel{\text{g Cl}_2} \times \frac{1\ \text{mol Cl}_2}{70.90\ \cancel{\text{g Cl}_2}} = 1.738\ \text{mol Cl}_2$$

Now that we have the quantity in moles, we can use the balanced chemical equation to construct a conversion factor that relates the number of moles of $Cl_2$ to the number of moles of $AlCl_3$. The numbers in the conversion factor come from the coefficients in the balanced chemical equation:

$$\frac{2\ \text{mol AlCl}_3}{3\ \text{mol Cl}_2}$$

Using this conversion factor with the molar quantity we calculated above, we get

$$1.738\ \cancel{\text{mol Cl}_2} \times \frac{2\ \text{mol AlCl}_3}{3\ \cancel{\text{mol Cl}_2}} = 1.159\ \text{mol AlCl}_3$$

So, we will get 1.159 mol of $AlCl_3$ if we react 123.2 g of $Cl_2$.

In this last example, we did the calculation in two steps. However, it is mathematically equivalent to perform the two calculations sequentially on one line:

$$123.2\ \cancel{\text{g Cl}_2} \times \frac{1\ \cancel{\text{mol Cl}_2}}{70.90\ \cancel{\text{g Cl}_2}} \times \frac{2\ \text{mol AlCl}_3}{3\ \cancel{\text{mol Cl}_2}} = 1.159\ \text{mol AlCl}_3$$

The units still cancel appropriately, and we get the same numerical answer in the end. Sometimes the answer may be slightly different from doing it one step at a time because of rounding of the intermediate answers, but the final answers should be effectively the same.

## EXAMPLE 10

How many moles of HCl will be produced when 249 g of $AlCl_3$ are reacted according to this chemical equation?

$$2AlCl_3 + 3H_2O(\ell) \rightarrow Al_2O_3 + 6HCl(g)$$

**Solution**

We will do this in two steps: convert the mass of $AlCl_3$ to moles and then use the balanced chemical equation to find the number of moles of HCl formed. The molar mass of $AlCl_3$ is 133.33 g/mol, which we have to invert to get the appropriate conversion factor:

$$249\ \cancel{\text{g AlCl}_3} \times \frac{1\ \text{mol AlCl}_3}{133.33\ \cancel{\text{g AlCl}_3}} = 1.87\ \text{mol AlCl}_3$$

Now we can use this quantity to determine the number of moles of HCl that will form. From the balanced chemical equation, we construct a conversion factor between the number of moles of $AlCl_3$ and the number of moles of HCl:

$$\frac{6\ \text{mol HCl}}{2\ \text{mol AlCl}_3}$$

Applying this conversion factor to the quantity of $AlCl_3$, we get

$$1.87\ \cancel{\text{mol AlCl}_3} \times \frac{6\ \text{mol HCl}}{2\ \cancel{\text{mol AlCl}_3}} = 5.61\ \text{mol HCl}$$

Alternatively, we could have done this in one line:

$$249\ \cancel{\text{g AlCl}_3} \times \frac{1\ \cancel{\text{mol AlCl}_3}}{133.33\ \cancel{\text{g AlCl}_3}} \times \frac{6\ \text{mol HCl}}{2\ \cancel{\text{mol AlCl}_3}} = 5.60\ \text{mol HCl}$$

The last digit in our final answer is slightly different because of rounding differences, but the answer is essentially the same.

***Test Yourself***

How many moles of $Al_2O_3$ will be produced when 23.9 g of $H_2O$ are reacted according to this chemical equation?

$$2AlCl_3 + 3H_2O(\ell) \rightarrow Al_2O_3 + 6HCl(g)$$

*Answer*

0.442 mol

A variation of the mole-mass calculation is to start with an amount in moles and then determine an amount of another substance in grams. The steps are the same but are performed in reverse order.

## EXAMPLE 11

How many grams of $NH_3$ will be produced when 33.9 mol of $H_2$ are reacted according to this chemical equation?

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

**Solution**

The conversions are the same, but they are applied in a different order. Start by using the balanced chemical equation to convert to moles of another substance and then use its molar mass to determine the mass of the final substance. In two steps, we have

$$33.9\ \cancel{\text{mol H}_2} \times \frac{2\ \text{mol NH}_3}{3\ \cancel{\text{mol H}_2}} = 22.6\ \text{mol NH}_3$$

Now, using the molar mass of $NH_3$, which is 17.03 g/mol, we get

$$22.6\ \cancel{\text{mol NH}_3} \times \frac{17.03\ \text{g NH}_3}{1\ \cancel{\text{mol NH}_3}} = 385\ \text{g NH}_3$$

***Test Yourself***

How many grams of $N_2$ are needed to produce 2.17 mol of $NH_3$ when reacted according to this chemical equation?

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

*Answer*

30.4 g (Note: here we go from a product to a reactant, showing that mole-mass problems can begin and end with any substance in the chemical equation.)

It should be a trivial task now to extend the calculations to **mass-mass calculations**, in which we start with a mass of some substance and end with the mass of another substance in the chemical reaction. For this type of calculation, the molar masses of two different substances must be used—be sure to keep track of which is which. Again, however, it is important to emphasize that before the balanced chemical reaction is used, the mass quantity must first be converted to moles. Then the coefficients of the balanced chemical reaction can be used to convert to moles of another substance, which can then be converted to a mass.

**mass-mass calculation**

A calculation in which you start with a given mass of a substance and calculate the mass of another substance involved in the chemical equation.

For example, let us determine the number of grams of $SO_3$ that can be produced by the reaction of 45.3 g of $SO_2$ and $O_2$:

$$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$$

First, we convert the given amount, 45.3 g of $SO_2$, to moles of $SO_2$ using its molar mass (64.06 g/mol):

$$45.3\ \cancel{\text{g SO}_2} \times \frac{1\ \text{mol SO}_2}{64.06\ \cancel{\text{g SO}_2}} = 0.707\ \text{mol SO}_2$$

Second, we use the balanced chemical reaction to convert from moles of $SO_2$ to moles of $SO_3$:

$$0.707\ \cancel{\text{mol SO}_2} \times \frac{2\ \text{mol SO}_3}{2\ \cancel{\text{mol SO}_2}} = 0.707\ \text{mol SO}_3$$

Finally, we use the molar mass of $SO_3$ (80.06 g/mol) to convert to the mass of $SO_3$:

$$0.707\ \cancel{\text{mol SO}_3} \times \frac{80.06\ \text{g SO}_3}{1\ \cancel{\text{mol SO}_3}} = 56.6\ \text{g SO}_3$$

We can also perform all three steps sequentially, writing them on one line as

$$45.3\ \cancel{\text{g SO}_2} \times \frac{1\ \cancel{\text{mol SO}_2}}{64.06\ \cancel{\text{g SO}_2}} \times \frac{2\ \cancel{\text{mol SO}_3}}{2\ \cancel{\text{mol SO}_2}} \times \frac{80.06\ \text{g SO}_3}{1\ \cancel{\text{mol SO}_3}} = 56.6\ \text{g SO}_3$$

We get the same answer. Note how the initial and all the intermediate units cancel, leaving grams of $SO_3$, which is what we are looking for, as our final answer.

## EXAMPLE 12

What mass of Mg will be produced when 86.4 g of K are reacted?

$$MgCl_2(s) + 2K(s) \longrightarrow Mg(s) + 2KCl(s)$$

**Solution**

We will simply follow the steps

$$\text{mass K} \longrightarrow \text{mol K} \longrightarrow \text{mol Mg} \longrightarrow \text{mass Mg}$$

In addition to the balanced chemical equation, we need the molar masses of K (39.09 g/mol) and Mg (24.31 g/mol). In one line,

$$86.4\ \cancel{\text{g K}} \times \frac{1\ \cancel{\text{mol K}}}{39.09\ \cancel{\text{g K}}} \times \frac{1\ \cancel{\text{mol Mg}}}{2\ \cancel{\text{mol K}}} \times \frac{24.31\ \text{g Mg}}{1\ \cancel{\text{mol Mg}}} = 26.87\ \text{g Mg}$$

***Test Yourself***

What mass of $H_2$ will be produced when 122 g of Zn are reacted?

$$Zn(s) + 2HCl(aq) \longrightarrow ZnCl_2(aq) + H_2(g)$$

*Answer*

3.77 g

## KEY TAKEAWAYS

- Mole quantities of one substance can be related to mass quantities using a balanced chemical equation.
- Mass quantities of one substance can be related to mass quantities using a balanced chemical equation.
- In all cases, quantities of a substance must be converted to moles before the balanced chemical equation can be used to convert to moles of another substance.

## EXERCISES

1. What mass of $CO_2$ is produced by the combustion of 1.00 mol of $CH_4$?

   $$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(\ell)$$

2. What mass of $H_2O$ is produced by the combustion of 1.00 mol of $CH_4$?

   $$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(\ell)$$

3. What mass of HgO is required to produce 0.692 mol of $O_2$?

   $$2HgO(s) \longrightarrow 2Hg(\ell) + O_2(g)$$

4. What mass of $NaHCO_3$ is needed to produce 2.659 mol of $CO_2$?

   $$2NaHCO_3(s) \longrightarrow Na_2CO_3(s) + H_2O(\ell) + CO_2(g)$$

5. How many moles of Al can be produced from 10.87 g of Ag?

   $$Al(NO_3)_3(s) + 3Ag \longrightarrow Al + 3AgNO_3$$

6. How many moles of HCl can be produced from 0.226 g of $SOCl_2$?

   $$SOCl_2(\ell) + H_2O(\ell) \longrightarrow SO_2(g) + 2HCl(g)$$

7. How many moles of $O_2$ are needed to prepare 1.00 g of $Ca(NO_3)_2$?

   $$Ca(s) + N_2(g) + 3O_2(g) \longrightarrow Ca(NO_3)_2(s)$$

8. How many moles of $C_2H_5OH$ are needed to generate 106.7 g of $H_2O$?

   $$C_2H_5OH(\ell) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(\ell)$$

9. What mass of $O_2$ can be generated by the decomposition of 100.0 g of $NaClO_3$?

   $$2NaClO_3 \longrightarrow 2NaCl(s) + 3O_2(g)$$

10. What mass of $Li_2O$ is needed to react with 1,060 g of $CO_2$?

    $$Li_2O(aq) + CO_2(g) \longrightarrow Li_2CO_3(aq)$$

11. What mass of $Fe_2O_3$ must be reacted to generate 324 g of $Al_2O_3$?

    $$Fe_2O_3(s) + 2Al(s) \longrightarrow 2Fe(s) + Al_2O_3(s)$$

12. What mass of Fe is generated when 100.0 g of Al are reacted?

$$Fe_2O_3(s) + 2Al(s) \rightarrow 2Fe(s) + Al_2O_3(s)$$

13. What mass of $MnO_2$ is produced when 445 g of $H_2O$ are reacted?

$$H_2O(\ell) + 2MnO_4^-(aq) + Br^-(aq) \rightarrow BrO_3^-(aq) + 2MnO_2(s) + 2OH^-(aq)$$

14. What mass of $PbSO_4$ is produced when 29.6 g of $H_2SO_4$ are reacted?

$$Pb(s) + PbO_2(s) + 2H_2SO_4(aq) \rightarrow 2PbSO_4(s) + 2H_2O(\ell)$$

15. If 83.9 g of ZnO are formed, what mass of $Mn_2O_3$ is formed with it?

$$Zn(s) + 2MnO_2(s) \rightarrow ZnO(s) + Mn_2O_3(s)$$

16. If 14.7 g of $NO_2$ are reacted, what mass of $H_2O$ is reacted with it?

$$3NO_2(g) + H_2O(\ell) \rightarrow 2HNO_3(aq) + NO(g)$$

17. If 88.4 g of $CH_2S$ are reacted, what mass of HF is produced?

$$CH_2S + 6F_2 \rightarrow CF_4 + 2HF + SF_6$$

18. If 100.0 g of $Cl_2$ are needed, what mass of NaOCl must be reacted?

$$NaOCl + HCl \rightarrow NaOH + Cl_2$$

**ANSWERS**

1. 44.0 g
3. $3.00 \times 10^2$ g
5. 0.0336 mol
7. 0.0183 mol
9. 45.1 g
11. 507 g
13. $4.30 \times 10^3$ g
15. 163 g
17. 76.7 g

# 5. YIELDS

**LEARNING OBJECTIVE**

1. **Define and determine theoretical yields, actual yields, and percent yields.**

In all the previous calculations we have performed involving balanced chemical equations, we made two assumptions: (1) the reaction goes exactly as written, and (2) the reaction proceeds completely. In reality, such things as side reactions occur that make some chemical reactions rather messy. For example, in the actual combustion of some carbon-containing compounds, such as methane, some CO is produced as well as $CO_2$. However, we will continue to ignore side reactions, unless otherwise noted.

The second assumption, that the reaction proceeds completely, is more troublesome. Many chemical reactions do not proceed to completion as written, for a variety of reasons (some of which we will consider in Chapter 13). When we calculate an amount of product assuming that all the reactant reacts, we calculate the **theoretical yield**, an amount that is theoretically produced as calculated using the balanced chemical reaction.

In many cases, however, this is not what really happens. In many cases, less—sometimes much less—of a product is made during the course of a chemical reaction. The amount that is actually produced in a reaction is called the **actual yield**. By definition, the actual yield is less than or equal to the theoretical yield. If it is not, then an error has been made.

Both theoretical yields and actual yields are expressed in units of moles or grams. It is also common to see something called a percent yield. The **percent yield** is a comparison between the actual yield and the theoretical yield and is defined as

$$\text{percent yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\%$$

It does not matter whether the actual and theoretical yields are expressed in moles or grams, as long as they are expressed in the same units. However, the percent yield always has units of percent. Proper

**theoretical yield**

An amount that is theoretically produced as calculated using the balanced chemical reaction.

**actual yield**

The amount that is actually produced in a chemical reaction.

**percent yield**

Actual yield divided by theoretical yield times 100% to give a percentage between 0% and 100%.

percent yields are between 0% and 100%—again, if percent yield is greater than 100%, an error has been made.

**EXAMPLE 13**

A worker reacts 30.5 g of Zn with nitric acid and evaporates the remaining water to obtain 65.2 g of $Zn(NO_3)_2$. What are the theoretical yield, the actual yield, and the percent yield?

$$Zn(s) + 2HNO_3(aq) \rightarrow Zn(NO_3)_2(aq) + H_2(g)$$

**Solution**

A mass-mass calculation can be performed to determine the theoretical yield. We need the molar masses of Zn (65.39 g/mol) and $Zn(NO_3)_2$ (189.41 g/mol). In three steps, the mass-mass calculation is

$$30.5\ \cancel{\text{g Zn}} \times \frac{1\ \cancel{\text{mol Zn}}}{65.39\ \cancel{\text{g Zn}}} \times \frac{1\ \cancel{\text{mol Zn(NO}_3)_2}}{1\ \cancel{\text{mol Zn}}} \times \frac{189.41\ \text{g Zn(NO}_3)_2}{1\ \cancel{\text{mol Zn(NO}_3)_2}} = 88.3\ \text{g Zn(NO}_3)_2$$

Thus, the theoretical yield is 88.3 g of $Zn(NO_3)_2$. The actual yield is the amount that was actually made, which was 65.2 g of $Zn(NO_3)_2$. To calculate the percent yield, we take the actual yield and divide it by the theoretical yield and multiply by 100:

$$\frac{65.2\ \text{g Zn(NO}_3)_2}{88.3\ \text{Zn(NO}_3)_2} \times 100\% = 73.8\%$$

The worker achieved almost three-fourths of the possible yield.

***Test Yourself***

A synthesis produced 2.05 g of $NH_3$ from 16.5 g of $N_2$. What is the theoretical yield and the percent yield?

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

*Answer*

theoretical yield = 20.1 g; percent yield = 10.2%

## Chemistry Is Everywhere: Actual Yields in Drug Synthesis and Purification

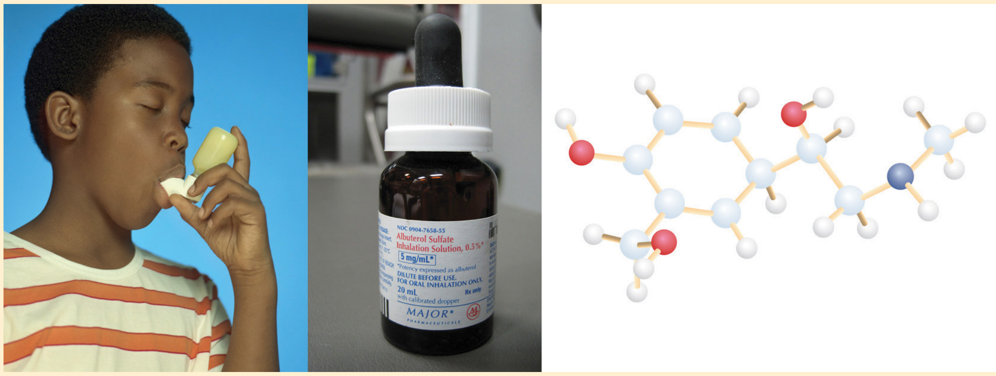
Many drugs are the product of several steps of chemical synthesis. Each step typically occurs with less than 100% yield, so the overall percent yield might be very small. The general rule is that the overall percent yield is the product of the percent yields of the individual synthesis steps. For a drug synthesis that has many steps, the overall percent yield can be very tiny, which is one factor in the huge cost of some drugs. For example, if a 10-step synthesis has a percent yield of 90% for each step, the overall yield for the entire synthesis is only 35%. Many scientists work every day trying to improve percent yields of the steps in the synthesis to decrease costs, improve profits, and minimize waste.

Even purifications of complex molecules into drug-quality purity are subject to percent yields. Consider the purification of impure albuterol. Albuterol ($C_{13}H_{21}NO_2$; accompanying figure) is an inhaled drug used to treat asthma, bronchitis, and other obstructive pulmonary diseases. It is synthesized from norepinephrine, a naturally occurring hormone and neurotransmitter. Its initial synthesis makes very impure albuterol that is purified in five chemical steps. The details of the steps do not concern us; only the percent yields do:

| | |
|---|---|
| impure albuterol → intermediate A | percent yield = 70% |
| intermediate A → intermediate B | percent yield = 100% |
| intermediate B → intermediate C | percent yield = 40% |
| intermediate C → intermediate D | percent yield = 72% |
| intermediate D → purified albuterol | percent yield = 35% |
| overall percent yield = 70% × 100% × 40% × 72% × 35% = 7.5% | |

That is, only about *one-fourteenth* of the original material was turned into the purified drug. This gives you one reason why some drugs are so expensive; a lot of material is lost in making a high-purity pharmaceutical.

A child using an albuterol inhaler, the container of albuterol medication, and a molecular model of the albuterol molecule.



*Source: Photo on far left © Thinkstock. Photo in center courtesy of Intropin, http://commons.wikimedia.org/wiki/File:Albuterol_Sulfate_%281%29.JPG.*

## KEY TAKEAWAYS

- Theoretical yield is what you calculate the yield will be using the balanced chemical reaction.
- Actual yield is what you actually get in a chemical reaction.
- Percent yield is a comparison of the actual yield with the theoretical yield.

## EXERCISES

1. What is the difference between the theoretical yield and the actual yield?
2. What is the difference between the actual yield and the percent yield?
3. A worker isolates 2.675 g of $SiF_4$ after reacting 2.339 g of $SiO_2$ with HF. What are the theoretical yield and the actual yield?

   $$SiO_2(s) + 4HF(g) \rightarrow SiF_4(g) + 2H_2O(\ell)$$

4. A worker synthesizes aspirin, $C_9H_8O_4$, according to this chemical equation. If 12.66 g of $C_7H_6O_3$ are reacted and 12.03 g of aspirin are isolated, what are the theoretical yield and the actual yield?

   $$C_7H_6O_3 + C_4H_6O_3 \rightarrow C_9H_8O_4 + HC_2H_3O_2$$

5. A chemist decomposes 1.006 g of $NaHCO_3$ and obtains 0.0334 g of $Na_2CO_3$. What are the theoretical yield and the actual yield?

   $$2NaHCO_3(s) \rightarrow Na_2CO_3(s) + H_2O(\ell) + CO_2(g)$$

6. A chemist combusts a 3.009 g sample of $C_5H_{12}$ and obtains 3.774 g of $H_2O$. What are the theoretical yield and the actual yield?

   $$C_5H_{12}(\ell) + 8O_2(g) \rightarrow 5CO_2 + 6H_2O(\ell)$$

7. What is the percent yield in Exercise 3?
8. What is the percent yield in Exercise 4?
9. What is the percent yield in Exercise 5?
10. What is the percent yield in Exercise 6?

## ANSWERS

1. Theoretical yield is what you expect stoichiometrically from a chemical reaction; actual yield is what you actually get from a chemical reaction.

3. theoretical yield = 4.052 g; actual yield = 2.675 g

5. theoretical yield = 0.635 g; actual yield = 0.0334 g

7. 66.02%

9. 5.26%

# 6. LIMITING REAGENTS

**LEARNING OBJECTIVES**

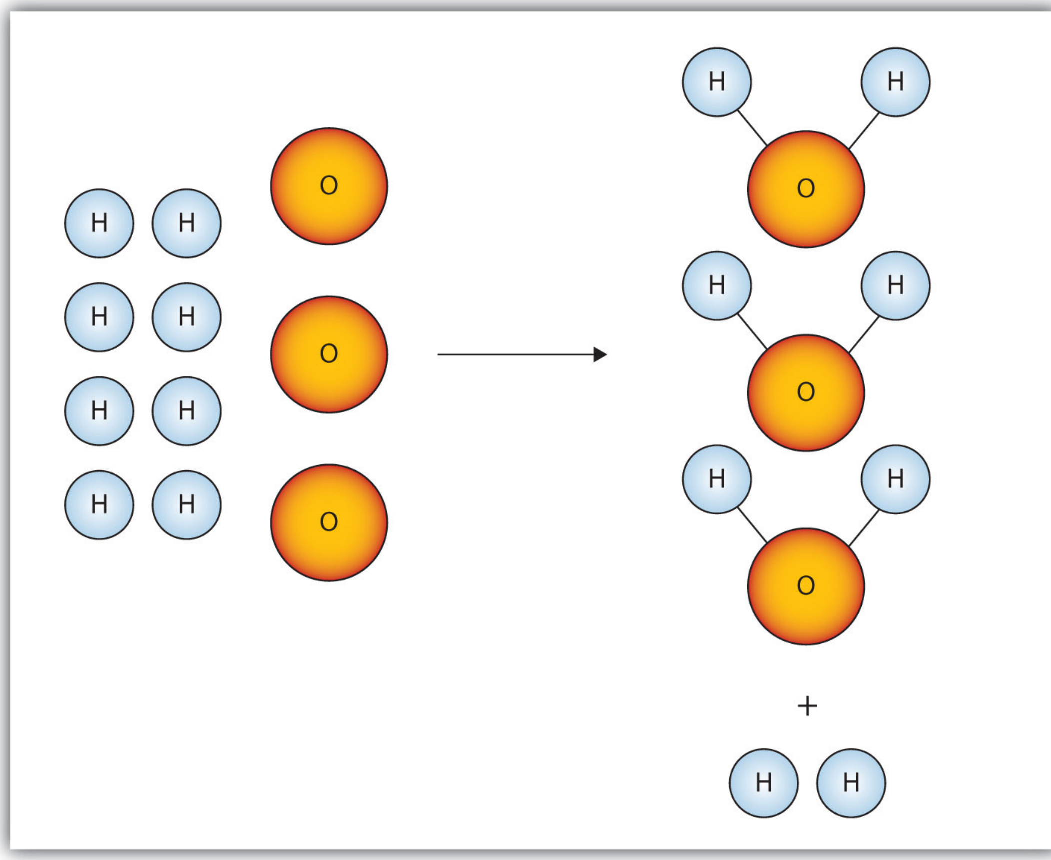
1. **Identify a limiting reagent from a set of reactants.**
2. **Calculate how much product will be produced from the limiting reagent.**
3. **Calculate how much reactant(s) remains when the reaction is complete.**

One additional assumption we have made about chemical reactions—in addition to the assumption that reactions proceed all the way to completion—is that all the reactants are present in the proper quantities to react to products. This is not always the case.

Consider Figure 5.2. Here we are taking hydrogen atoms and oxygen atoms (left) to make water molecules (right). However, there are not enough oxygen atoms to use up all the hydrogen atoms. We run out of oxygen atoms and cannot make any more water molecules, so the process stops when we run out of oxygen atoms.

**FIGURE 5.2** **Making Water**

In this scenario for making water molecules, we run out of O atoms before we use up all the H atoms. Similar situations exist for many chemical reactions when one reactant runs out before the other.




**limiting reagent**

The reactant that runs out first.

A similar situation exists for many chemical reactions: you usually run out of one reactant before all of the other reactant has reacted. The reactant you run out of is called the **limiting reagent**; the other reactant or reactants are considered to be *in excess*. A crucial skill in evaluating the conditions of a chemical process is to determine which reactant is the limiting reagent and which is in excess.

The key to recognizing which reactant is the limiting reagent is based on a mole-mass or mass-mass calculation: whichever reactant gives the *lesser* amount of product is the limiting reagent. What we need to do is determine an amount of one product (either moles or mass) assuming all of each reactant reacts. Whichever reactant gives the least amount of that particular product is the limiting reagent. It does not matter which product we use, as long as we use the same one each time. It does not matter whether we determine the number of moles or grams of that product; however, we will see shortly that knowing the final mass of product can be useful.

For example, consider this reaction:

$$4As(s) + 3O_2(g) \rightarrow 2As_2O_3(s)$$

Suppose we start a reaction with 50.0 g of As and 50.0 g of $O_2$. Which one is the limiting reagent? We need to perform two mole-mass calculations, each assuming that each reactant reacts completely. Then we compare the amount of the product produced by each and determine which is less.

The calculations are as follows:

$$50.0\ \cancel{g\ As} \times \frac{1\ \cancel{mol\ As}}{74.92\ \cancel{g\ As}} \times \frac{2\ mol\ As_2O_3}{4\ \cancel{mol\ As}} = 0.334\ mol\ As_2O_3$$

$$50.0\ \cancel{g\ O_2} \times \frac{1\ \cancel{mol\ O_2}}{32.00\ \cancel{g\ O_2}} \times \frac{2\ mol\ As_2O_3}{3\ \cancel{mol\ O_2}} = 1.04\ mol\ As_2O_3$$

Comparing these two answers, it is clear that 0.334 mol of $As_2O_3$ is less than 1.04 mol of $As_2O_3$, so arsenic is the limiting reagent. If this reaction is performed under these initial conditions, the arsenic will run out before the oxygen runs out. We say that the oxygen is "in excess."

Identifying the limiting reagent, then, is straightforward. However, there are usually two associated questions: (1) what mass of product (or products) is then actually formed? and (2) what mass of what reactant is left over? The first question is straightforward to answer: simply perform a conversion from the number of moles of product formed to its mass, using its molar mass. For $As_2O_3$, the molar mass is 197.84 g/mol; knowing that we will form 0.334 mol of $As_2O_3$ under the given conditions, we will get

$$0.334\ \cancel{mol\ As_2O_3} \times \frac{197.84\ g\ As_2O_3}{1\ \cancel{mol\ As_2O_3}} = 66.1\ g\ As_2O_3$$

The second question is somewhat more convoluted to answer. First, we must do a mass-mass calculation relating the limiting reagent (here, As) to the other reagent ($O_2$). Once we determine the mass of $O_2$ that reacted, we subtract that from the original amount to determine the amount left over. According to the mass-mass calculation,

$$50.0\ \cancel{g\ As} \times \frac{1\ \cancel{mol\ As}}{74.92\ \cancel{g\ As}} \times \frac{3\ \cancel{mol\ O_2}}{4\ \cancel{mol\ As}} \times \frac{32.00\ g\ O_2}{1\ \cancel{mol\ O_2}} = 16.0\ g\ O_2\ \text{reacted}$$

Because we reacted 16.0 g of our original $O_2$, we subtract that from the original amount, 50.0 g, to get the mass of $O_2$ remaining:

$$50.0\ g\ O_2 - 16.0\ g\ O_2\ \text{reacted} = 34.0\ g\ O_2\ \text{left over}$$

You must remember to perform this final subtraction to determine the amount remaining; a common error is to report the 16.0 g as the amount remaining.

## EXAMPLE 14

A 5.00 g quantity of Rb are combined with 3.44 g of $MgCl_2$ according to this chemical reaction:

$$2Rb(s) + MgCl_2(s) \rightarrow Mg(s) + 2RbCl(s)$$

What mass of Mg is formed, and what mass of what reactant is left over?

**Solution**

Because the question asks what mass of magnesium is formed, we can perform two mass-mass calculations and determine which amount is less.

$$5.00\ \cancel{\text{g Rb}} \times \frac{1\ \cancel{\text{mol Rb}}}{85.47\ \cancel{\text{g Rb}}} \times \frac{1\ \cancel{\text{mol Mg}}}{2\ \cancel{\text{mol Rb}}} \times \frac{24.31\ \text{g Mg}}{1\ \cancel{\text{mol Mg}}} = 0.711\ \text{g Mg}$$

$$3.44\ \cancel{\text{g MgCl}_2} \times \frac{1\ \cancel{\text{mol MgCl}_2}}{95.21\ \cancel{\text{g MgCl}_2}} \times \frac{1\ \cancel{\text{mol Mg}}}{1\ \cancel{\text{mol MgCl}_2}} \times \frac{24.31\ \text{g Mg}}{1\ \cancel{\text{mol Mg}}} = 0.878\ \text{g Mg}$$

The 0.711 g of Mg is the lesser quantity, so the associated reactant—5.00 g of Rb—is the limiting reagent. To determine how much of the other reactant is left, we have to do one more mass-mass calculation to determine what mass of $MgCl_2$ reacted with the 5.00 g of Rb and then subtract the amount reacted from the original amount.

$$5.00\ \cancel{\text{g Rb}} \times \frac{1\ \cancel{\text{mol Rb}}}{85.47\ \cancel{\text{g Rb}}} \times \frac{1\ \cancel{\text{mol MgCl}_2}}{2\ \cancel{\text{mol Rb}}} \times \frac{95.21\ \text{g Mg}}{1\ \cancel{\text{mol MgCl}_2}} = 2.78\ \text{g MgCl}_2\ \text{reacted}$$

Because we started with 3.44 g of $MgCl_2$, we have

$$3.44\ \text{g MgCl}_2 - 2.78\ \text{g MgCl}_2\ \text{reacted} = 0.66\ \text{g MgCl}_2\ \text{left}$$

***Test Yourself***

Given the initial amounts listed, what is the limiting reagent, and what is the mass of the leftover reagent?

$$\underset{22.7\ \text{g}}{MgO(s)} + \underset{17.9\ \text{g}}{H_2S(g)} \rightarrow MgS(s) + H_2O(\ell)$$

*Answer*

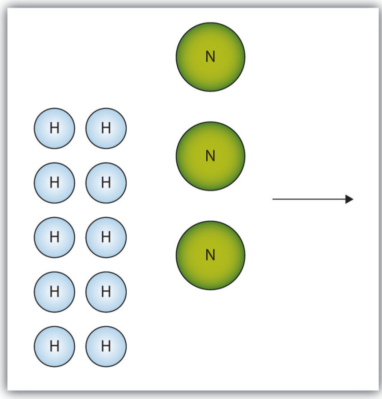
$H_2S$ is the limiting reagent; 1.5 g of MgO are left over.

## KEY TAKEAWAYS

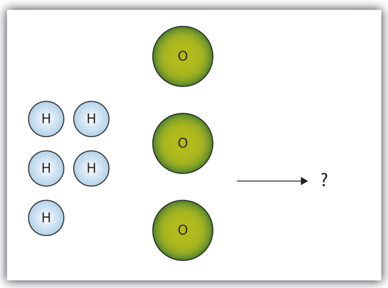
- The limiting reagent is that reactant that produces the least amount of product.
- Mass-mass calculations can determine how much product is produced and how much of the other reactants remain.

### EXERCISES

1. The box below shows a group of nitrogen and hydrogen molecules that will react to produce ammonia, $NH_3$. What is the limiting reagent?




2. The box below shows a group of hydrogen and oxygen molecules that will react to produce water, $H_2O$. What is the limiting reagent?




3. Given the statement "20.0 g of methane is burned in excess oxygen," is it obvious which reactant is the limiting reagent?
4. Given the statement "the metal is heated in the presence of excess hydrogen," is it obvious which substance is the limiting reagent despite not specifying any quantity of reactant?
5. Acetylene ($C_2H_2$) is formed by reacting 7.08 g of C and 4.92 g of $H_2$.

   $$2C(s) + H_2(g) \rightarrow C_2H_2(g)$$

   What is the limiting reagent? How much of the other reactant is in excess?
6. Ethane ($C_2H_6$) is formed by reacting 7.08 g of C and 4.92 g of $H_2$.

   $$2C(s) + 3H_2(g) \rightarrow C_2H_6(g)$$

   What is the limiting reagent? How much of the other reactant is in excess?
7. Given the initial amounts listed, what is the limiting reagent, and how much of the other reactant is in excess?

   $$\underset{35.6\text{ g}}{P_4O_6(s)} + \underset{4.77\text{ g}}{6H_2O(\ell)} \rightarrow 4H_3PO_4$$

8. Given the initial amounts listed, what is the limiting reagent, and how much of the other reactant is in excess?

   $$\underset{377\text{ g}}{3NO_2(g)} + \underset{244\text{ g}}{H_2O(\ell)} \rightarrow 2HNO_3(aq) + NO(g)$$

9. To form the precipitate $PbCl_2$, 2.88 g of NaCl and 7.21 g of $Pb(NO_3)_2$ are mixed in solution. How much precipitate is formed? How much of which reactant is in excess?
10. In a neutralization reaction, 18.06 g of KOH are reacted with 13.43 g of $HNO_3$. What mass of $H_2O$ is produced, and what mass of which reactant is in excess?

### ANSWERS

1. Nitrogen is the limiting reagent.
3. Yes; methane is the limiting reagent.
5. C is the limiting reagent; 4.33 g of $H_2$ are left over.
7. $H_2O$ is the limiting reagent; 25.9 g of $P_4O_6$ are left over.
9. 6.06 g of $PbCl_2$ are formed; 0.33 g of NaCl is left over.

## 7. END-OF-CHAPTER MATERIAL

### ADDITIONAL EXERCISES

1. How many molecules of $O_2$ will react with $6.022 \times 10^{23}$ molecules of $H_2$ to make water? The reaction is $2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$.

2. How many molecules of $H_2$ will react with $6.022 \times 10^{23}$ molecules of $N_2$ to make ammonia? The reaction is $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$.
3. How many moles are present in 6.411 kg of $CO_2$? How many molecules is this?
4. How many moles are present in 2.998 mg of $SCl_4$? How many molecules is this?
5. What is the mass in milligrams of $7.22 \times 10^{20}$ molecules of $CO_2$?
6. What is the mass in kilograms of $3.408 \times 10^{25}$ molecules of $SiS_2$?
7. What is the mass in grams of 1 molecule of $H_2O$?
8. What is the mass in grams of 1 atom of Al?
9. What is the volume of 3.44 mol of Ga if the density of Ga is 6.08 g/mL?
10. What is the volume of 0.662 mol of He if the density of He is 0.1785 g/L?
11. For the chemical reaction

$$2C_4H_{10}(g) + 13O_2(g) \rightarrow 8CO_2(g) + 10H_2O(\ell)$$

assume that 13.4 g of $C_4H_{10}$ reacts completely to products. The density of $CO_2$ is 1.96 g/L. What volume in liters of $CO_2$ is produced?

12. For the chemical reaction

$$2GaCl_3(s) + 3H_2(g) \rightarrow 2Ga(\ell) + 6HCl(g)$$

if 223 g of $GaCl_3$ reacts completely to products and the density of Ga is 6.08 g/mL, what volume in milliliters of Ga is produced?

13. Calculate the mass of each product when 100.0 g of CuCl react according to the reaction

$$2CuCl(aq) \rightarrow CuCl_2(aq) + Cu(s)$$

What do you notice about the sum of the masses of the products? What concept is being illustrated here?

14. Calculate the mass of each product when 500.0 g of $SnCl_2$ react according to the reaction

$$2SnCl_2(aq) \rightarrow SnCl_4(aq) + Sn(s)$$

What do you notice about the sum of the masses of the products? What concept is being illustrated here?

15. What mass of $CO_2$ is produced from the combustion of 1 gal of gasoline? The chemical formula of gasoline can be approximated as $C_8H_{18}$. Assume that there are 2,801 g of gasoline per gallon.
16. What mass of $H_2O$ is produced from the combustion of 1 gal of gasoline? The chemical formula of gasoline can be approximated as $C_8H_{18}$. Assume that there are 2,801 g of gasoline per gallon.
17. A chemical reaction has a theoretical yield of 19.98 g and a percent yield of 88.40%. What is the actual yield?
18. A chemical reaction has an actual yield of 19.98 g and a percent yield of 88.40%. What is the theoretical yield?
19. Given the initial amounts listed, what is the limiting reagent, and how much of the other reactants are in excess?

$$\underset{35.0\ g}{P_4} + \underset{12.7\ g}{3NaOH} + \underset{9.33\ g}{3H_2O} \rightarrow 2Na_2HPO_4 + PH_3$$

20. Given the initial amounts listed, what is the limiting reagent, and how much of the other reactants are in excess?

$$\underset{46.3\ g}{2NaCrO_2} + \underset{88.2\ g}{3NaBrO_4} + \underset{32.5\ g}{2NaOH} \rightarrow 3NaBrO_3 + 2Na_2CrO_4 + H_2O$$

21. Verify that it does not matter which product you use to predict the limiting reagent by using both products in this combustion reaction to determine the limiting reagent and the amount of the reactant in excess. Initial amounts of each reactant are given.

$$\underset{26.3\text{ g}}{C_3H_8} + \underset{21.8\text{ g}}{5O_2} \rightarrow 3CO_2(g) + 4H_2O(\ell)$$

22. Just in case you suspect Exercise 21 is rigged, do it for another chemical reaction and verify that it does not matter which product you use to predict the limiting reagent by using both products in this combustion reaction to determine the limiting reagent and the amount of the reactant in excess. Initial amounts of each reactant are given.

$$\underset{35.0\text{ g}}{2P_4} + \underset{12.7\text{ g}}{6NaOH} + \underset{9.33\text{ g}}{6H_2O} \rightarrow 3Na_2HPO_4 + 5PH_3$$

## ANSWERS

1. $1.2044 \times 10^{24}$ molecules
3. 145.7 mol; $8.77 \times 10^{25}$ molecules
5. 52.8 mg
7. $2.99 \times 10^{-23}$ g
9. 39.4 mL
11. 20.7 L
13. 67.91 g of $CuCl_2$; 32.09 g of Cu. The two masses add to 100.0 g, the initial amount of starting material, demonstrating the law of conservation of matter.
15. 8,632 g
17. 17.66 g
19. The limiting reagent is NaOH; 21.9 g of $P_4$ and 3.61 g of $H_2O$ are left over.
21. Both products predict that $O_2$ is the limiting reagent; 20.3 g of $C_3H_8$ are left over.